Economic Production Inventory Model with Remanufacturing of Defective Item in Two Cases using Hexagonal Fuzzy Number

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Abstract—*For economic and environment reasons, defective items are reworked to become serviceable again. In this paper, our proposed model is considered in two cases. In case (i), the defective items are remanufacturing during the production process. In case (ii), the defective items returned from customers are also remanufacturing per cycle. The rate of remanufacturing items are considered as a random variable, which follows a hyper exponential distribution. Hexagonal fuzzy number is defined and the properties are given. Expected total is derived and the parameters involved in this total cost is represented by hexagonal fuzzy number. Our proposed model is defuzzified by Mean Deviation Method. Maximum inventory level is obtained by using MATLAB Software.*

Keywords*: Inventory, Defective items, Remanufacturing, Hyper Exponential Distribution, Hexagonal Fuzzy Number, Mean Deviation Method.*

AMS Subject Classification(2010): 90B05

1. INTRODUCTION

Inventory planning and control is concerned with the acquisition and storage of the materials required for supporting various business operations. In classical model such as EOQ and EPQ developed [5] and [13] , it is assumed that the rate of replenishment /production and unit price of an item are constant. But later theorists worked out more comprehensive, realistic market – sensitive models. Multi - item classical inventory models are presented in well known books [4], [12], [14]., etc. In [3], the EOQ model with demand dependent unit cost using geometric programming technique is given. In[10] Inventory models in both constrained and unconstrained situation are considered.

While modeling an inventory problem, it is assumed that demand and various relevant costs are defined with certainty. But, in real life, demand and various relevant costs are not exactly known. In this situation, uncertainties are treated as randomness and are handled through probability theory. In certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory, originally introduced in [15] and may be applied. In [2] fuzzy optimization through aggregation operations that combine fuzzy goals and fuzzy -decision space is explained. Introduction of fuzzy arithmetic operations are given by[1]. Later, the fuzzy linear programming model was formulated and an approach for solving problem in linear programming model with fuzzy numbers has been presented in [16].

Generally the inventory model are formulated by considering that only the perfect items are produced. However, in reality production items may not always be perfect. So, a proportion of the produced items can be found to be defective. A continuous production control inventory model for deteriorating items with shortages is developed in[11]. The EPQ model was investigated by considering production of various types of non-perfect products is explained in[6]. The economic order quantity model for deteriorating items with two level of trade credit in one replacement cycle is developed in[9]. A multi-objective imperfect quality inventory model with defective items solved by modified geometric programming approach is formulated in[8]. The concept of Hyper-exponential distribution is given in [7].

In recent time, power scarcity has affected the large scale industries in manufacturing goods, to solve this problem, solar plants are being installed in many forms. It incurs a cost. The cost as alternative power supply cost, its operating and maintenance cost. This paper refers, defective items are reworked to become serviceable again. The defective item is considered as a random variable which follows Hyper exponential distribution. Also the hexagonal fuzzy number is defined and its properties are given. So the multi – item inventory model by using hexagonal fuzzy number with alternative power supply, its operating and maintenance costs has been considered.

The parameters involved in this paper are assumed to be imprecise in nature and are represented by hexagonal fuzzy number. Expected total cost for our proposed model is derived and using Mean Deviation method, the model is deffuzified. Maximum inventory level and Expected total cost are obtained by using MATLAB software.

2. ASSUMPTIONS AND NOTATIONS

Our proposed model is constructed under the following assumptions and notations.

Assumptions:

1. Production rate is finite.
2. Shortages are not allowed.
3. Lead time is zero.
4. Time horizon is infinite.
5. Alternative power supply (solar plants, their operating and maintenance) costs are allowed.
6. Demand rate is random variable, it follows Normal Distribution during the period$[0,t_2]$.
7. Demand rate is uniform during the period $[t_2, t_3]$.
8. Remanufacturing of defective items is done during the period $[t_1,t_2]$
9. Rate of remanufacturing items is a random variable, which follows. Hyper -exponential distribution.

Notations: The following notations for the i[th] item (i=1,2,3…n)

$\tilde{d}_i^1$ - fuzzy demand rate.

d_i - demand rate (random variable).

$\tilde{k}_i$ - fuzzy production rate.

$\tilde{\theta}_i$ - rate of fuzzy defective items.

$\tilde{\gamma}_i$ - rate of fuzzy unrecoverable items.

$R_r (= \tilde{\theta}_i - \tilde{\gamma}_i)$ - rate of remanufacturing items(random variable).

$\tilde{s}_{ei}$ - fuzzy operating and maintenance cost of solar plants per cycle.

W_t - working time of solar plants per cycle.

$\tilde{H}_i$ - fuzzy holding cost per unit per unit time.

$\tilde{s}_i$ - fuzzy setup cost per cycle.

S_E - fixed solar plants cost for plan period.

c_{ui} - fuzzy unrecoverable items cost for plan period.

I_{mi} - Maximum inventory level at time t_2.

I_{si} - Inventory level at time t_1.

TC - Total cost.

Hyper-exponential distribution

A random variable 'X' is said to follow Hyper-exponential distribution with two parameters a>0, b>0, the probability density function is given by

$$f(t) = \beta a e^{-at} + (1-\beta) b e^{-bt} \ , \quad 0 \le \beta \le 1.$$

Such a system which has two kinds of components a proportion 'β' of components having high failure rate a and the remaining proportion (1-β) of components having a lower failure rate b.

3. FORMULATION OF THE CRISP MODEL

To derive the inventory cost function, divide the time interval $[0,t_3]$ into three parts: $[0,t_1]$, $[t_1,t_2]$ and $[t_2,t_3]$. The regular production started at time t=0 and stops at time $t=t_1$. During $[0,t_1]$, the inventory level gradually increases and some items are defective. The regular production is stopped at t_1. During the period $[t_1,t_2]$, remanufacturing process is done. So, stock builds up during the period $[0, t_2]$ and declines during the period $[t_2 , t_3]$ .The stock is reduced to zero at t_3.

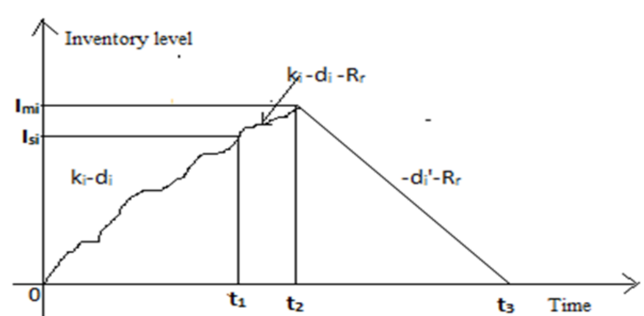

Fig:1 Production inventory model with remanufacturing of defective items

Let $I_i(t)$ be the inventory level during the period $[0,t_3]$. Then the differential equations governing the instantaneous state of $I_i(t)$ at any time t are given by,

$$\frac{dI_i(t)}{dt}+\theta_i I_i(t)=k_i-E(d_i),\quad 0\le t\le t_1 \qquad \text{--------(1)}$$

$$\frac{dI_i(t)}{dt}+\theta_i I_i(t)=k_i-E(d_i)-E(R_r),\quad t_1\le t\le t_2 \qquad \text{-------(2)}$$

$$\frac{dI_i(t)}{dt}+\theta_i I_i(t)=-d_i^{'}-E(R_r),\quad t_2\le t\le t_3 \qquad \text{------(3)}$$

The boundary conditions are

$$I_i(0)=0,\ I_i(t_1)=I_{si},\ I_i(t_2)=I_{mi},\ I_i(t_3)=0 \qquad (4)$$

The expected number of rate of remanufacturing item is

$$E(R_r)=\int_0^{t_2}(\theta_i-\gamma_i)f_x(t)dt$$

$$=\int_0^{t_2}(\theta_i-\gamma_i)\left(\beta a e^{-at}+(1-\beta)be^{-bt}\right)dt$$

$$E(R_r)=(\theta_i-\gamma_i)\left[1-\beta e^{-at_2}-(1-\beta)e^{-bt_2}\right] \qquad (5)$$

The expected number of demand during the period $[0,t_2]$ is

$$E\{d_i\}=\frac{1}{\sigma\sqrt{2\pi}}\int_0^{t_2}d_i e^{-\frac{1}{2}\left(\frac{d_i-\mu}{\sigma}\right)^2}dd_i$$

$$E\{d_i\}=\frac{1}{\sqrt{2\pi}}\left[\frac{t_2 e^{-\frac{1}{2}\left(\frac{\mu-t_2}{\sigma}\right)^2}}{\left(\frac{\mu-t_2}{\sigma}\right)}\right]$$

The solutions of equations (1)-(4) are given by

$$I_i(t)=\begin{cases}\left[\frac{k_i-E(d_i)}{\theta_i}\right]\left(1-e^{-\theta_i t}\right) & 0\le t\le t_1\\ \left[\left\{\left(\frac{(k_i-E(d_i)-E(R_r))}{\theta_i}\right)\left(1-e^{-\theta_i(t-t_1)}\right)+I_{si}e^{-\theta_i(t-t_1)}\right\}\right] & t_1\le t\le t_2\\ \left[\frac{d_i^{'}+E(R_r)}{\theta_i}\right]\left(e^{\theta_i(t_3-t)}\right) & t_2\le t\le t_3\end{cases}$$

------------(6)

Using (4) in (6), I_{si} and I_{mi} are derived.

$$I_{si}=\left[\frac{k_i-E(d_i)}{\theta_i}\right]\left(1-e^{-\theta_i t_1}\right) \qquad (7)$$

$$I_{mi}=\left\{\left(\frac{k_i-E(d_i)-E(R_r)}{\theta_i}\right)\left(1-e^{-\theta_i(t_2-t_1)}\right)+I_{si}e^{-\theta_i(t_2-t_1)}\right\} \text{-----} \qquad (8)$$

The excepted inventory holding cost $=H_i\int_0^{t_3}I_i(t)dt$

$$=H_i\left[\int_0^{t_1}I_i(t)dt+\int_{t_1}^{t_2}I_i(t)dt+\int_{t_2}^{t_3}I_i(t)dt\right]$$

$$=H_i\left[\begin{array}{l}\int_0^{t_1}\left[\frac{k_i-E(d_i)}{\theta_i}\right]\left(1-e^{-\theta_i t}\right)dt+\int_{t_1}^{t_2}\left[\left\{\frac{(k_i-E(d_i)-E(R_r))}{\theta_i}\left(1-e^{-\theta_i(t-t_1)}\right)+I_{si}e^{-\theta_i(t-t_1)}\right\}\right]dt\\ +\int_{t_2}^{t_3}\left[\frac{d_i+E(R_r)}{\theta_i}\right]\left(e^{\theta_i(t_3-t)}\right)dt\end{array}\right]$$

$$=H_i\left[\begin{array}{l}\left(\frac{k_i-E(d_i)}{\theta_i}\right)\left(t_1+\frac{e^{-\theta_i t_1}-1}{\theta_i}\right)+\left(\frac{k_i-E(d_i)-E(R_r)}{\theta_i}\right)\left(t_2+\frac{e^{-\theta_i(t_2-t_1)}-1}{\theta_i}-t_1\right)\\ +\left(\frac{1-e^{-\theta_i(t_2-t_1)}}{\theta_i}\right)\left(\frac{k_i-E(d_i)}{\theta_i}\right)\left(1-e^{-\theta_i t_1}\right)-\left(\frac{d_i^{'}+E(R_r)}{\theta_i}\right)\left(\frac{1-e^{\theta_i(t_3-t_2)}}{\theta_i}\right)\end{array}\right]$$

Setup cost=s_i

Solar plant operating and maintenance cost=$s_{ei}W_t$

Unrecoverable defective item cost=$c_{ui}\gamma_i$

Solar Plant cost = S_E

Expected total cost = Expected holding cost + Setup cost+Solar plants operating and maintenan cost +unrecoverable defective item cost+ Solar plants cost.

Expected total cost is given by

$$E(TC)=\sum_{i=1}^{n}\left(\frac{1}{t_3}\right)\left[\begin{array}{l}H_i\left[\begin{array}{l}\left(\frac{k_i-E(d_i)}{\theta_i}\right)\left(t_1+\frac{e^{-\theta_i t_1}-1}{\theta_i}\right)+\left(\frac{k_i-E(d_i)-E(R_r)}{\theta_i}\right)\left(t_2+\frac{e^{-\theta_i(t_2-t_1)}-1}{\theta_i}-t_1\right)\\ +\left(\frac{1-e^{-\theta_i(t_2-t_1)}}{\theta_i}\right)\left(\frac{k_i-E(d_i)}{\theta_i}\right)\left(1-e^{-\theta_i t_1}\right)-\left(\frac{d_i+E(R_r)}{\theta_i}\right)\left(\frac{1-e^{\theta_i(t_3-t_2)}}{\theta_i}\right)\end{array}\right]\\ +s_i+s_{ei}W_t+c_{ui}\gamma_i\end{array}\right]+S_E \qquad (9)$$

4. HEXAGONAL FUZZY NUMBER AND ITS PROPERTIES

A Hexagonal fuzzy number $\widetilde{A}$ is described as a fuzzy subset on the real line R whose membership function $\mu_{\widetilde{A}}(x)$ is defined as follows

$$\mu_{\widetilde{A}}(x)=\begin{cases}0 & for\quad x\le a\\ W_A\left(\frac{x-a}{b-a}\right) & for\quad a\le x\le b\\ W_A+(1-W_A)\left(\frac{x-b}{c-b}\right) & for\quad b\le x\le c\\ 1 & for\quad c\le x\le d\\ W_A+(1-W_A)\left(\frac{c-x}{c-d}\right) & for\quad d\le x\le e\\ W_A\left(\frac{f-x}{f-e}\right) & for\quad e\le x\le f\\ 0 & for\quad x\ge f\end{cases}$$

Where $0.6 \le W_A < 1$, a, b, c, d, e and f are real numbers.

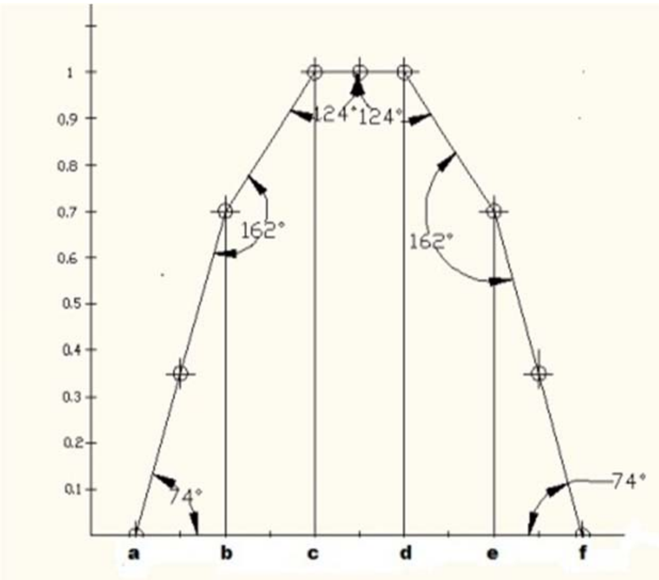

Fig. 2: Graphical Representation of Hexagonal Fuzzy Number

This type of fuzzy number is denoted by $\widetilde{A} = (a,b,c,d,e,f;W_A)_{HFN}$

$\mu_{\widetilde{A}}$ **satisfies the following conditions:**

1. $\mu_{\widetilde{A}}$ is a continuous mapping from R to the closed interval $[0,1]$.
2. $\mu_{\widetilde{A}}$ is a convex function.
3. $\mu_{\widetilde{A}}(x) = 0, -\infty < x \le a$.
4. $\mu_{\widetilde{A}}(x) = \delta_l(x)$ is strictly increasing on (a , c)
5. $\mu_{\widetilde{A}}(x) = 1, x \in [c,d]$.
6. $\mu_{\widetilde{A}}(x) = \delta_r(x)$ is strictly decreasing on (d ,f)
7. $\mu_{\widetilde{A}}(x) = 0, f < x \le \infty$.

Remark:

If $0 < W_A < 0{\cdot}6$, then $\widetilde{A}$ becomes a trapezoidal fuzzy number.

5. MEAN DEVIATION OF THE FUZZY NUMBER:

Let us consider $\widetilde{A}$ is a Hexagonal fuzzy number $\widetilde{A} = (a,b,c,d,e,f)_{HFN}$, then the procedure of Mean deviation of the Hexagonal fuzzy number is explained as follows.

The membership function is divided into two parts. One at the left of $x_{m1}, x_l(\alpha)$ and the other at right of $x_{m2}, x_r(\alpha)$.

The quantity $\delta_l(\widetilde{A}) = \int_{\alpha=0}^{1} [x_m - x_l(\alpha)] d\alpha$ is called the left mean deviation and the quantity $\delta_r(\widetilde{A}) = \int_{\alpha=0}^{1} [x_r(\alpha) - x_m] d\alpha$ is called the right mean deviation and $\delta(\widetilde{A}) = \delta_l(\widetilde{A}) + \delta_r(\widetilde{A})$ is called the mean deviation of the fuzzy number.

By construction, a fuzzy number have $x_l(\alpha) \le x_{m1} \le x_m \le x_{m2} \le x_r(\alpha)$, $\delta_l(\widetilde{A}) \ge 0, \delta_r(\widetilde{A}) \ge 0, \delta(\widetilde{A}) \ge 0$. Also $\delta_l(\widetilde{A})$ represents the area to the left of x_m and $\delta_r(\widetilde{A})$ represents the area of the right. Where $\delta(\widetilde{A})$ is the sum of the two areas.

Therefore the mean deviation of the Hexagonal fuzzy number is

$$\delta(\widetilde{A}) = \begin{bmatrix} \left(\dfrac{W_A}{2(a-b)}\right)\left\{(x_m-b)^2-(x_m-a)^2\right\} + \left(\dfrac{1-W_A}{2(b-c)}\right)\left\{(x_m-c)^2-(x_m-b)^2\right\} \\ +\left(\dfrac{W_A}{2(e-f)}\right)\left\{(e-x_m)^2-(f-x_m)^2\right\} + \left(\dfrac{1-W_A}{2(d-e)}\right)\left\{(d-x_m)^2-(e-x_m)^2\right\} \end{bmatrix}$$

6. THE PROPOSED INVENTORY MODEL IN FUZZY ENVIRONMENT

If the cost parameters are fuzzy number, then the problem (9) is transformed to

$$E(\widetilde{T}C) = \sum_{i=1}^{n} \left[\left(\frac{1}{t_3}\right) \left(\widetilde{H}_i \left[\begin{array}{l} \left(\dfrac{\widetilde{k}_i - E(d_i)}{\widetilde{\theta}_i}\right)\left(t_1 + \dfrac{e^{-\widetilde{\theta}_i t_1} - 1}{\widetilde{\theta}_i}\right) + \left(\dfrac{\widetilde{k}_i - E(d_i) - E(R_r)}{\widetilde{\theta}_i}\right)\left(t_2 + \dfrac{e^{-\widetilde{\theta}_i (t_2 - t_1)} - 1}{\widetilde{\theta}_i} - t_1\right) \\ + \left(\dfrac{1 - e^{-\widetilde{\theta}_i (t_2 - t_1)}}{\widetilde{\theta}_i}\right)\left(\dfrac{\widetilde{k}_i - E(d_i)}{\widetilde{\theta}_i}\right)\left(1 - e^{-\widetilde{\theta}_i t_1}\right) - \left(\dfrac{\widetilde{d}'_i + E(R_r)}{\widetilde{\theta}_i}\right)\left(\dfrac{1 - e^{\widetilde{\theta}_i (t_3 - t_2)}}{\widetilde{\theta}_i}\right) \end{array} \right] + \widetilde{s}_i + \widetilde{s}_{ei} W_t + \widetilde{c}_{ui} \widetilde{\gamma}_i \right) \right] + S_E$$

(10)

Where ~ represents the fuzzification of the parameters.

In the proposed model , the parameters d_i, k_i, θ_i , γ_i ,H_i, s_i, s_{ei} and c_{ui} are considered as Hexagonal fuzzy number.

$\widetilde{d}_i = [d_1, d_2, d_3, d_4, d_5, d_6]$ $\widetilde{k}_i = [k_1, k_2, k_3, k_4, k_5, k_6]$

$\widetilde{\theta}_i = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6]$

$\widetilde{\gamma}_i = [\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6]$

$\widetilde{H}_i = [H_1, H_2, H_3, H_4, H_5, H_6]$

$\widetilde{s}_i = [s_1, s_2, s_3, s_4, s_5, s_6]$

$\widetilde{s}_{ei} = [s_{e1}, s_{e2}, s_{e3}, s_{e4}, s_{e5}, s_{e6}]$

$\widetilde{c}_{ui} = [c_{u1}, c_{u2}, c_{u3}, c_{u4}, c_{u5}, c_{u6}]$

The corresponding fuzzy problem (10) is

$$E(\widetilde{TC})=\sum_{i=1}^{n}\left[\left(\frac{1}{t_3}\right)\left(\delta_{\widetilde{H}i}\left[\begin{array}{l}\left(\frac{\delta_{\widetilde{k}i}-\delta_{E(d_i)}}{\delta_{\widetilde{\theta}_i}}\right)\left(t_1+\frac{e^{-\delta_{\theta_i}t}-1}{\delta_{\widetilde{\theta}i}}\right)+\left(\frac{\delta_{\widetilde{k}i}-\delta_{E(d_i)}-E(R_r)}{\delta_{\widetilde{\theta}i}}\right)\left(t_2+\frac{e^{-\delta_{\widetilde{\theta}i}(t_2-t_1)}-1}{\delta_{\widetilde{\theta}i}}-t_1\right)\\+\left(\frac{1-e^{-\delta_{\widetilde{\theta}_i}(t_2-t_1)}}{\delta_{\widetilde{\theta}i}}\right)\left(\frac{\delta_{\widetilde{k}i}-E(d_i)}{\delta_{\widetilde{\theta}i}}\right)\left(1-e^{-\delta_{\widetilde{\theta}i}t_1}\right)-\left(\frac{\delta'_{\widetilde{d}i}+E(R_r)}{\delta_{\widetilde{\theta}i}}\right)\left(\frac{1-e^{\delta_{\widetilde{\theta}i}(t_3-t_2)}}{\delta_{\widetilde{\theta}i}}\right)\end{array}\right]\\+\delta_{\widetilde{s}i}+\delta_{\widetilde{s}ei}W_t+\delta_{c_{ui}}\delta_{\widetilde{\gamma}i}\right)\right]+S_E \quad -(11)$$

where

$$\delta_{\widetilde{H}_i}=\left[\begin{array}{l}\left(\frac{W_A}{2(H_1-H_2)}\right)\left\{(x_{H_m}-H_2)^2-(x_{H_m}-H_1)^2\right\}+\left(\frac{1-W_A}{2(H_2-H_3)}\right)\left\{(x_{H_m}-H_3)^2-(x_{H_m}-H_2)^2\right\}\\+\left(\frac{W_A}{2(H_5-H_6)}\right)\left\{(H_5-x_{H_m})^2-(H_6-x_{H_m})^2\right\}+\left(\frac{1-W_A}{2(H_4-H_5)}\right)\left\{(H_4-x_{H_m})^2-(H_5-x_{H_m})^2\right\}\end{array}\right]$$

Similarly for $\delta_{\widetilde{m}_i}, \delta_{\widetilde{s}_i}, \delta_{\widetilde{s}_{ei}}, \delta_{\widetilde{k}_i}, \delta_{\widetilde{d}_i}, \delta_{\widetilde{c}_{ui}}, \delta_{\widetilde{\gamma}_i}$ and $\delta_{\widetilde{\theta}_i}$.

7. NUMERICAL EXAMPLE

The steel items manufacturing company produces two items . The relevant data for the two items are given below and using MATAB software, the maximum inventory level and Expected total cost are obtained .

$\widetilde{k}_1$ = [10000,11000,12000,13000,14000,15000]

$\widetilde{d}_1$ = [500,600,700,800,900,1000] $\widetilde{s}_{e1}$ = [7,8,9,10,11,12]

$\widetilde{H}_i$ = [12,14,16,18,20,22] , $\widetilde{c}_{u1}$ = [1,2,3,4,5,6]

$\widetilde{s}_1$ = [200,300,400,500,600,700] ,

Table1: Comparison of the crisp and fuzzy results

Model	Item	ki	di	θ_i & γ_i	Hi	si	sei	cui	a, b & β	μ & σ	I^*_{si}	I^*_{mi}	Total optimal quantity	E(TC) (Rs)	Expenditure of one unit cost (Rs)
crisp	1	10000	500	0.700 0.001	12	200	7	1	8 5 0.6	6.4 1.2	5525	7620	8416	1044900	124
	2	500	50	0.610 0.001	10	100	5	1			569	796			
	1	13000	800	0.760 0.040	18	500	10	4	5 2 0.5	16 1.2	5814	8372	9239	1065000	115
	2	1300	80	0.670 0.004	13	400	8	1.3			595	867			
fuzzy	1	3400	340	0.068 0.034	6.8	340	3.4	3.4	8 5 0.6	6.4 1.2	7979	12080	13288	1485200	112
	2	340	34	0.068 0.003	3.4	340	3.4	0.34	5 2 0.5	16 1.2	798	1208			

$\widetilde{\theta}_1$ = [0.7,0.72,0.74,0.76,0.78,0.80]

$\widetilde{\gamma}_1$ = [0.01,0.02,0.03,0.04,0.05,0.06]

$\widetilde{k}_2$ = [1000,1100,1200,1300,1400,1500]

$\widetilde{d}_2$ = [50,60,70,80,90,100], $\widetilde{H}_2$ = [10,11,12,13,14,15]

$\widetilde{s}_2$ = [100,200,300,400,500,600], $\widetilde{s}_{e2}$ = [5,6,7,8,9,10]

$\widetilde{c}_{u2}$ = [1,1.1,1.2,1.3,1.4,1.5]

$\widetilde{\gamma}_2$ = [0.001,0.002,0.003,0.004,0.005,0.006]

S_E=1000000, W_A=0.7, W_t=1200.

8. CONCLUSION

From table 1, observed that the optimal values are given for the fuzzy models along with the crisp model. The fuzzy values of maximum inventory level and inventory level at t_1 are high when compare with crisp values and also the crisp values of minimum average total cost is high when compare to fuzzy model. Finally we conclude that these model can be executable in the real world.

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